Name: Student number

Computational Science 260

Midterm Exam

Fill in answers in space provided. Use back of page for draft.

Oct. 27

Marks

1. Use a truth table to prove that $(P \wedge Q_1) \vee (\neg P \wedge Q_2)$ is logically equivalent to $(P \Rightarrow Q_1) \wedge (\neg P \Rightarrow Q_2)$. 15

P	Q_1	Q_2	$P \wedge Q_1$	$\neg P \wedge Q_2$	$(P \wedge Q_1) \vee (\neg P \wedge Q_2)$	$P \Rightarrow Q_1$	$\neg P \Rightarrow Q_2$	$(P \Rightarrow Q_1) \wedge (\neg P \Rightarrow Q_2)$
T	T	T	T	F	T	[illegible]	T	T
T	T	F	T	F	T	[illegible]	T	T
T	[illegible]	T	F	F	F	F	T	F
T	F	F	F	F	F	T	T	F
F	T	T	F	T	T	T	T	T
[illegible]	T	F	F	F	F	T	F	F
[illegible]	[illegible]	[illegible]	F	T	T	T	T	T
[illegible]	[illegible]	[illegible]	F	F	F	T	F	F

Same, therefore equivalent.

2. Children dance at nursery school, and each child has exactly one partner. Let $P(x, y)$ be true of x is the partner of y, or if y is the partner of x. Express the fact that each child has exactly one partner in predicate calculus. 12

$$\forall x \, \exists y \, (P(x,y) \wedge \forall z \, (P(x,z) \Rightarrow x = y))$$

3. Given $\forall y(P(y) \vee Q(y))$ and $\exists x \neg P(x)$, give a derivation to show $\exists x Q(x)$. 14

$\forall y\,(P(y) \vee Q(y)),\ \exists x\, \neg P(x) \vdash \exists x\, Q(x)$

1.	$\forall y\,(P(y) \vee Q(y))$	Premise
2.	$\exists x\, \neg P(x)$	Premise
3.	$\neg P(a)$	2, EI
4.	$P(a) \vee Q(a)$	1, S^{y}_{x}
5.	$Q(a)$	3, 4, D.S.
6.	$\exists x\, Q(x)$	5, EG

4. Let P stand for "The new year starts October 21", Q for "4 is even" and R for "Canada is a tropical country". Assign the appropriate truth values to all these propositions. Translate $(P \wedge Q) \vee (Q \Rightarrow R)$ into English. Moreover, find the truth value of this expression. 12

The new year start in Oct. 25 and 4 is even, or 4 is even implies Canada is a tropical country

P : The new year starts Oct. 25 : F

Q : 4 is even T

R : Canada is a tropical country F

2

$\underbrace{\underset{F}{(P \wedge Q)} \vee \underset{F}{(Q \Rightarrow R)}}_{F}$ Statement false

5. Consider the following Prolog data base 15

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abc(X,Y) :- cde(X,U), efg(V,U), hij(V,Y).
cde(a,b).
cde(a,c).
efg(d,b).
efg(h,c).
hij(h,b).
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TRACE

Suppose the query is abc(a,b). Trace the execution of the query abc(a,b). The trace should indicate in which order the different goals are attempted, together with an indication whether or not they succeed. Use S for succeed and F for fail.

abc(a,b)

cde(a,b) S
efg(d,b) S
hij(d,b) F
efg(V,b) F
cde(a,c) S
efg(h,c) S
hij(h,b) S
abc(a,b) S

6. In a Prolog data base, there is a fact for each English word, indicating whether it is a noun, a verb, an article, and so on. For instance, there is a fact noun(dog) to indicate that "dog" is a noun, there is a fact verb(run) to indicate that "run" is a verb, and there is a fact article(the) to indicate that "the" is an article. Design a rule sentence(X, Y, Z) which must succeed of X is an article, Y is a noun, 10

Prolog

and Z is a verb.

sentence (X, Y, Z) :- article (X), noun (Y), verb (Z),

7. Let A be a set, and let $\#A$ be the number of elements in the set. Show that $\#(A \vee B) \leq \#A + \#B$. Moreover, give an example where $\#(A \vee B) = \#A + \#B$. 10

Elements appearing in both A and B are counted twice in $\#A + \#B$, and only once in $\#(A \vee B)$.

$A = \{1, 2, 3\}$ $B = \{4\}$

$A \cup B = \{1, 2, 3, 4\}$.

8. Let $f(n) = 2 - f(n-1)/2$, with $f(0) = 0$. Find $f(3)$ by replacing $f(m)$ with a proper expression. 12

$$f(3) = 2 - \frac{f(2)}{2}$$

$$= 2 - \frac{2 - f(1)/2}{2}$$

$$= 2 - \frac{2 - (2 - f(0)/2)/2}{2}$$

$$= 2 - \frac{2 - (2)/2}{2} = 1\tfrac{1}{2}$$

100

——————— The End ———————